Analysis

PSU Math Relays 2016

- There are **27** problems.

- For each problem, place your answer in the appropriate blank of the answer sheet provided.

- Simplify each answer as much as possible. Rationalize fractions. Use $+\infty$ and $-\infty$ for positive and negative infinity. Give numerical answers in fractional form, if applicable. Do not use decimal approximations.

- No calculators are allowed on the exam.

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1. Find the limit $\lim\limits_{x \to 5} x^3 - 3x^2 + 100$.

2. Find the limit $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.

3. Find the limit $\lim\limits_{x \to \infty} \dfrac{x^2 + \sin(x)}{5x^2 + x\cos(x)}$.

4. Find the limit $\lim\limits_{x \to -\infty} \dfrac{x\sin(1/x)}{x + 1}$

5. Find the limit $\lim\limits_{x \to 0} \dfrac{1}{x^2} - \dfrac{1}{\sin^2(x)}$

   In problems 6-8, let $f(x) = 5x^3 - 10x^2 + 3$.

6. $f(1) =?$

7. $f'(1) =?$

8. $f''(1) =?$

   In problems 9-11, let $g(x) = \ln(x^2 + 1)$.

9. $g(0) =?$

10. $g'(0) =?$

11. $g''(0) =?$

12. If y is a function of x defined implicitly by the equation $\sin(y) = \cos(x)$, find $\dfrac{dy}{dx}$ at the point $(\pi/2, \pi)$.

13. Find the x-coordinate of the intersection of the tangent line to $y = \ln(x)$ at $x = e$ with the x-axis.

For problems 14 and 15, let $f(x) = x^3 - 9x^2 + 24x + 12$. Use open intervals (a, b) for your answer.

14. Find the interval(s) on which f is increasing.

15. Find the interval(s) on which f is concave down.

16. If $f(x) = x^7 \cos^6(\theta)$, find $f^{(7)}(0)$.

17. Evaluate $\int_0^2 x^4 - x^2 + 1 \, dx$. Write your answer as a single fraction.

18. Evaluate $\int_\pi^{3\pi/2} \cos(x) \, dx$.

19. Evaluate $\int_0^{\pi/4} \tan(x) \, dx$. Write your answer in the form $\ln(a)$.

20. Evaluate $\int_{\frac{1-e^2}{3}}^{\frac{1-e}{3}} \frac{1}{1 - 3x} \, dx$.

21. Evaluate $\int_0^1 \frac{x}{1 + x} \, dx$.

22. Evaluate $\int_0^1 \frac{3x^2 + 2x}{1 + x} \, dx$.

23. Evaluate $\int_{-\infty}^0 \frac{e^x}{1 + e^{2x}} \, dx$.

24. Evaluate $\int_0^{\pi/2} \sin^3(\theta) \cos^2(\theta) \, d\theta$.

25. Evaluate $\int_0^1 x^2 e^x \, dx$.

26. Evaluate $\int_1^e t^4 \ln(t) \, dt$.

27. Find the point on the upper half of the unit circle whose tangent line intersects the x-axis at $x = 3$.