

Algebraic Simplification

PSU Math Relays 2017

- For each problem below choose the best answer from the given choices and shade the appropriate letter on the answer sheet.
- You must shade your choice on the answer sheet. Only the answer sheet will be graded.
- Calculator is **not** allowed.

1. Perform the operation $(3x^2 - x - 7)(x^2 - 5)$.
(A) $3x^4 - x^3 - 22x^2 - 35$ (B) $3x^4 - 15x^3 + 5x + 35$ (C) $3x^4 - x^3 - 5x + 35$
(D) $3x^4 - x^3 - 22x^2 + 5x + 35$ (E) none of these
2. If $f(x) = \sqrt{x} - x$, then $f(9)$ is
(A) 6 (B) 3 (C) 0 (D) -3 (E) -6
3. If $g(x) = 2x - x^2$, then $g(g(-2))$ is
(A) -80 (B) -40 (C) 0 (D) 20 (E) 40
4. Simplify $\sqrt[3]{-\frac{27}{64}}$.
(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{4}i$ (D) $-\frac{3}{4}i$ (E) $-\frac{9}{8}$
5. Simplify $\sqrt{\sqrt[3]{x}}$.
(A) $\sqrt[5]{x}$ (B) $\sqrt[3]{x^2}$ (C) $\sqrt[6]{x}$ (D) $\sqrt[6]{x^5}$ (E) $\sqrt[4]{x}$
6. Simplify $\frac{x^{1/3}}{x^{1/4}}$.
(A) $x^{1/12}$ (B) $x^{7/12}$ (C) $x^{1/7}$ (D) $x^{-1/12}$ (E) x
7. Simplify $\sqrt{8} + 6\sqrt{18} - 3\sqrt{50}$.
(A) $3\sqrt{2}$ (B) $\sqrt{10}$ (C) $-2\sqrt{2}$ (D) $5\sqrt{2}$ (E) none of these
8. Simplify: $\frac{x+y}{x^{-1}+y^{-1}}$.
(A) xy (B) $x-y$ (C) -1 (D) x^2+y^2 (E) $(x+y)^2$

9. Simplify the expression $[(x^2y^{-5}z^5)^{\frac{1}{3}}]^{-12}$ and write the answer so that it contains only positive exponents.
- (A) $\frac{x^8z^{20}}{y^{20}}$ (B) $\frac{1}{x^8y^{20}z^{20}}$ (C) $\frac{y^{20}}{x^8z^{20}}$ (D) $\frac{x^6z^9}{y^9}$ (E) $\frac{y^9}{x^6z^9}$
10. Let $-4 < x < 2$. Simplify $|2x + 9| - |3x - 7|$.
- (A) $-x + 16$ (B) $5x + 2$ (C) $5x + 16$ (D) $x - 2$ (E) none of these
11. Simplify $\frac{\sqrt[3]{16x^{-5}y^7z^9}}{\sqrt[3]{250xy^{10}z^{-6}}}$.
- (A) $\frac{z^5}{5x^2y}$ (B) $\frac{2z^5}{5x^2y^3}$ (C) $\frac{z^3}{10x^3y}$ (D) $\frac{2z^5}{5x^2y}$ (E) none of these
12. Simplify $\frac{x^2 - 6x + 9}{x^2 - x - 6} \div \frac{x^2 - 7x + 12}{x^2 - 2x - 8}$.
- (A) $\frac{(x-3)^2}{(x-4)^2}$ (B) $\frac{x-3}{x-4}$ (C) $\frac{x+3}{x-3}$ (D) $\frac{x+3}{x+2}$ (E) none of these
13. Rationalize the denominator of $\frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}}$.
- (A) 1 (B) -29 (C) $6 + \sqrt{35}$ (D) $6 - \sqrt{35}$ (E) $2 - \sqrt{35}$
14. Simplify $\frac{12!}{(2!)(3!)(10!)}$. Here $n!$ represents the factorial of n .
- (A) 1 (B) 11 (C) 22 (D) $\frac{1}{6}$ (E) none of these
15. Simplify i^{115} .
- (A) 1 (B) i (C) -1 (D) $-i$ (E) $115i$
16. Simplify $\frac{3 - 4i}{2 + 3i}$
- (A) $\frac{6}{5} + \frac{17}{5}i$ (B) $\frac{-6}{13} - \frac{17}{13}i$ (C) $\frac{-6}{13} + \frac{17}{13}i$ (D) $\frac{-6}{13} - \frac{17}{13}i$ (E) $1 - 7i$
17. $\frac{x^3 - 4x^2 - 3}{x + 2} =$
- (A) $x^2 - 6x + 12 - \frac{27}{x + 2}$ (B) $x^3 - 6x^2 + 2x - \frac{27}{x + 2}$
- (C) $x - 6 + \frac{9}{x + 2}$ (D) $x^2 - 6x + \frac{9}{x + 2}$ (E) $x^2 - 2x - 4 - \frac{11}{x + 2}$

18. Let $f(x) = -x^2 + 5$ and $g(x) = x - 3$. Find the composite function $f(g(x))$.
- (A) $-x^2 - 6x - 4$ (B) $-x^2 + 14$ (C) $-x^2 + 6x - 4$
 (D) $-x^3 + 3x^2 + 5x - 15$ (E) $-x^2 + 6x + 14$
19. If $a + b = 2$ and $a^2 + b^2 = 7$, find $a^3 + b^3$.
- (A) 9 (B) 14 (C) 17 (D) 41 (E) 49
20. Simplify $\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + \cdots + \frac{1}{\log_{100} N}$.
- (A) $\frac{1}{\log_{5050} N}$ (B) $\frac{1}{\log_{100!} N}$ (C) $\frac{1}{\log_N 100!}$ (D) $\log_{5050} N$ (E) $\log_{100!} N$
21. Simplify $\sqrt[3]{\sqrt{5} - 2} - \sqrt[3]{\sqrt{5} + 2}$.
- (A) -4 (B) -3 (C) -2 (D) -1 (E) none of these
22. Evaluate the determinant $\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix}$.
- (A) 5 (B) -5 (C) 11 (D) -11 (E) 24
23. Evaluate the determinant $\begin{vmatrix} -2 & 7 & 5 \\ 3 & -1 & 0 \\ 6 & 0 & -4 \end{vmatrix}$.
- (A) -54 (B) -10 (C) 0 (D) 72 (E) 106
24. Find the product of the matrices $\begin{pmatrix} -5 & 0 \\ -3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ -5 & 7 \end{pmatrix}$.
- (A) $\begin{pmatrix} -15 & 0 \\ -29 & 28 \end{pmatrix}$ (B) $\begin{pmatrix} -15 & 0 \\ 15 & 28 \end{pmatrix}$ (C) $\begin{pmatrix} -15 & 0 \\ 43 & -21 \end{pmatrix}$
 (D) $\begin{pmatrix} -15 & 0 \\ 4 & 28 \end{pmatrix}$ (E) none of these
25. Let $A = \begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}$. The inverse matrix $A^{-1} =$
- (A) $\begin{pmatrix} -2 & 4 \\ -3 & 5 \end{pmatrix}$ (B) $\begin{pmatrix} 1/5 & -1/4 \\ 1/3 & -1/2 \end{pmatrix}$ (C) $\frac{1}{\begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}}$
 (D) $\begin{pmatrix} -5 & 4 \\ -3 & 2 \end{pmatrix}$ (E) $\begin{pmatrix} -1 & 2 \\ -1.5 & 2.5 \end{pmatrix}$