

Analysis

PSU Math Relays 2017

- There are 27 problems
 - For each problem, place your answer in the appropriate blank of the answer sheet provided.
 - Simplify each answer as much as possible. Rationalize fractions. Use $+\infty$ and $-\infty$ for positive and negative infinity. Give numerical answers in fractional form, if applicable. Do not use decimal approximations.
 - No calculators are allowed on the exam.
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1. Find the limit $\lim_{x \rightarrow 4} 3 + x^{\frac{1}{2}}$
2. Find the limit $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(3x)}$
3. Find the limit $\lim_{x \rightarrow -1} \frac{3x^2 + 4x + 1}{x + 1}$
4. Find the limit $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 20x}}{10x - 2}$
5. Find the limit $\lim_{x \rightarrow \infty} \frac{|x| + x}{x + 1}$
6. Find the limit $\lim_{x \rightarrow \infty} \frac{\ln x}{x^{\frac{1}{2}}}$

Complete the square in problems 1-2 to put the expression in the form $a(x + b)^2 + c$.

7. $x^2 + 8x + 4$
8. $4x^2 + 24x + 7$

Let $f(x) = 5x^7 - 3x^5 + 12x^2 - 17x + 9$.

9. $f(1) =$
10. $f'(1) =$
11. $f''(1) =$

Let $g(x) = \sin(3x) - 1$.

12. $g(0) =$
13. $g'(0) =$
14. $g''(0) =$

Calculate the derivate of $f(x)$ in problems 15-18.

15. $f(x) = 3x^6 - 9x^3 + 18x^2 - 17$

16. $f(x) = \tan x$

17. $f(x) = e^x$

18. $f(x) = \ln x$

19. State the interval(s) where the function $y = 2x^3 + 15x^2 - 84x + 17$ is increasing.

20. State the interval(s) where the function $y = 2x^3 + 15x^2 - 84x + 17$ is concave down.

Calculate the following integrals

21. Evaluate $\int \sqrt{x+1} \, dx$.

22. Evaluate $\int_{\pi/4}^{\pi/3} \sin x \, dx$.

23. Evaluate $\int \frac{3}{3x+1} \, dx$.

24. Evaluate $\int_0^1 x^2 e^x \, dx$.

25. Evaluate $\int 5x(x^4 + 7) \, dx$.

26. Evaluate $\int \cos^2 x \, dx$.

27. Evaluate $\frac{d}{dx} \int_5^1 2t^3 \sqrt{t^8 + t + 7} \, dt$.