Marathon–2017

You may use a calculator. **Do not write on the test below but only on the plain paper provided.** Answers put on the form below will not be graded.

1. Let $p(x) = 3 + 2(x-1) + 2(x-1)(x-2)$.
   (a) Calculate $p(1)$.
   (b) Calculate $p(2)$.
2. Let $L(x) = \frac{3}{2}(1-x) + \frac{5}{2}(x+1)$.
   (a) Calculate $L(-1)$.
   (b) Calculate $L(1)$.
3. Let $p(x) = 1 + (x-1) + (x-1)(x-2) + (x-1)(x-2)(x-3)$.
   (a) Calculate $p(1)$.
   (b) Calculate $p(2)$.
   (c) Calculate $p(3)$.
   (d) Construct a function $q(x)$ so that $q(1) = 1$, $q(2) = 2$, and $q(3) = 3$.
4. Let $L(x) = \frac{A}{b-a}(b-x) + \frac{B}{b-a}(x-a)$.
   (a) Calculate $L(a)$.
   (b) Calculate $L(b)$.
   (c) Construct a function $L(x)$ so that $L(-\pi) = -1$ and $L(\pi) = 1$.
5. Let $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{N}{x_n}\right)$ for $n = 0, 1, 2, \ldots$.
   (a) Let $N = 5$ and $x_0 = 3$. Find x_1.
   (b) Let $N = 5$ and $x_0 = 3$. Find x_2.
   (c) Let $N = 5$ and $x_0 = 3$. Find x_3.
   (d) Solve $L = \dfrac{1}{2}\left(L + \dfrac{N}{L}\right)$ for L in terms of N.
   (e) What is the value of $\lim\limits_{n\to\infty} x_n$ where x_n is as defined originally?
6. Define the binary operation $\wedge$ by

$\wedge$	0	1
0	0	0
1	0	1

   . Given 4-bit binary numbers $x_4x_3x_2x_1$ and $y_4y_3y_2y_1$ define $x_4x_3x_2x_1 \wedge y_4y_3y_2y_1$ to be $z_4z_3z_2z_1$ where $z_i = x_i \wedge y_i$ for $i = 1, 2, 3, 4$.
   (a) Calculate $1100 \wedge 0110$.
   (b) Calculate $x_4x_3x_2x_1 \wedge 0100$.
7. Given a binary number $x_4x_3x_2x_1$ define $x_4x_3x_2x_1 << 1$ to be equal to $x_3x_2x_1 0$.
   (a) Calculate $1011 << 1$.
   (b) With $\wedge$ as defined in an earlier problem, calculate $x_4x_3x_2x_1 \wedge (1 << 1)$.
   (c) Defining $x_4x_3x_2x_1 << 2$ as $(x_4x_3x_2x_1 << 1) << 1$, calculate $x_4x_3x_2x_1 \wedge (10 << 2)$.