Notation and Definitions:

- $\gcd(a, b)$ means the *greatest common divisor* of the positive integers a and b.

- $\operatorname{lcm}(a, b)$ means the *least common multiple* of the positive integers a and b.

- If $\gcd(a, b) = 1$, we say a and b are *relatively prime*.

- $\varphi(n)$ is *Euler's phi function*, which is the number of positive integers less than or equal to n which are relatively prime to n.

- $a \equiv b \pmod{m}$ if $a - b$ is an integer multiple of m.

- $\lceil x \rceil$ denotes the "ceiling" of x, the smallest integer greater than or equal to x.

- $\lfloor x \rfloor$ denotes the "floor" of x, the greatest integer less than or equal to x.

- $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ denotes the binomial coefficient.

Instructions: Write your answer on the indicated line of the answer sheet **exactly as directed** in each problem. Only the answer sheet will be graded.

1. List all positive divisors of 246 in **increasing order**.

2. How many positive divisors does $2^4 \cdot 3^7 \cdot 7^2$ have?

3. Please find $\gcd(240, 56)$.

4. Please find $\operatorname{lcm}(56, 24)$.

5. What is the exponent on 2 in the prime factorization of 528?

6. What is the exponent on 5 in the prime factorization of 750?

7. Suppose that a, b are positive integers and we know that $\gcd(a, b) = 3$ and $a \cdot b = 540$. Please find $\operatorname{lcm}(a, b)$.

8. Please find the 19th prime.

Questions 9-11 refer to the following definition of a, b:
Suppose that a, b have prime factorizations
$$a = 2^3 \cdot 3^6 \cdot 5^1$$
$$b = 2^5 \cdot 3^8 \cdot 11^2$$

9. What is the exponent on 2 in the prime factorization of $\gcd(a, b)$?

10. What is the exponent on 3 in the prime factorization of $\operatorname{lcm}(a, b)$?

11. What is the exponent on 11 in the prime factorization of $\operatorname{lcm}(a, b)$?

12. What is the smallest possible positive value for c satisfying the equation below, assuming that x, y are arbitrary integers?
$$6x + 21y = c.$$

13. Please find $\gcd(4532, 324)$.

14. Which of the numbers below is divisible by 3? **Write either "x" or "y" on the answer sheet.**

$$x = 12,342,342$$
$$y = 342,428$$

15. Which of the numbers below is divisible by 4? **Write either "x" or "y" on the answer sheet.**

$$x = 498,234$$
$$y = 423,424$$

16. Please calculate $\lceil \sqrt{60} \rceil$.

17. Please calculate $\lfloor \sqrt{122} \rfloor$.

18. Please calculate $\varphi(17)$.

19. Please calculate $\varphi(121)$.

20. Please calculate $\varphi(24)$.

21. Please calculate $\binom{20}{2}$.

22. What is the coefficient of $a^2 b^5$ in the simplified expansion of $(a+b)^7$?

23. Please calculate:
$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}.$$

24. Please convert the base 2 (binary) number 1010101 to base 10.

25. Please convert the base 10 number 123 to base 4.

26. What values of $x \in \{0,1,2,3,4,5\}$ satisfy $x + 2 \equiv 1 \pmod 6$?

27. What values of $x \in \{0,1,2,3,\ldots 9\}$ satisfy $2x \equiv 4 \pmod{10}$?

28. What values of $x \in \{0,1,2,3,\ldots 8\}$ satisfy $x^2 \equiv 7 \pmod 9$?

29. Please calculate $82^{2016} \pmod 9$ (**answer as an integer x, $0 \le x \le 8$**)

30. Please calculate $131^{2015} \pmod{11}$ (**answer as an integer x, $0 \le x \le 10$**)

Thank you for participating in the Pittsburg State Math Relays!