Analysis

PSU Math Relays 2018

- For each problem place your answer in the appropriate blank on the answer sheet provided.
- Simplify each answer as far as possible. Write numerical answers in exact form, such as fractions or radicals, rather than decimal approximations.
- You may **not** use a calculator on this test.

In problems 1–4 find the indicated limit.

1. $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - 5x + 6}$
2. $\lim\limits_{x \to \infty} \dfrac{-2x^2 + 5}{3x^2 - 2x + 1}$
3. $\lim\limits_{\theta \to 0} \dfrac{\tan 3\theta}{4\theta}$
4. $\lim\limits_{x \to 0^+} f(x)$, where $f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ 3x - 1 & \text{if } x > 0 \end{cases}$

In problems 5–7 Let $f(x) = x^{3/2}$.

5. $f(9) = ?$
6. $f'(0) = ?$
7. $f^{(3)}(4) = ?$

In problems 8–12 find the indicated derivative.

8. $y = \sqrt{x^2 - 6x + 5}$, $\dfrac{dy}{dx} = ?$
9. $f(t) = \tan(3t)$, $f'(t) = ?$
10. $f(x) = te^{t^2 - 3t}$, $f'(t) = ?$
11. $g(x) = \ln(x^2 + 2)$, $g'(x) = ?$
12. $y = \displaystyle\int_0^{\cos x} \frac{1}{\sqrt{1 - t^2}}\, dt$ with $0 < x < \pi$, $\dfrac{dy}{dx} = ?$

13. Find the slope-intercept form of the equation for the tangent line to the curve defined by the function $y = \sqrt{x}$, at $x = 4$.

In problems 14–19 let $f(x) = x^5 - 20x + 1$. Use the interval notation (a, b) to write intervals in your answers.

14. Find the interval(s) on which f is increasing.
15. Find the interval(s) on which f is decreasing.
16. Find the interval(s) on which f is concave up.
17. Find the interval(s) on which f is concave down.
18. Find the x value(s) at which f has a local maximum.
19. Find the x value(s) at which f has a local minimum.

20. Find the absolute maximum value of the function $f(x) = \sin x - \cos x$ on the interval $[0, 2\pi]$.

In problems 21–24 evaluate the indicated integral.

21. $\displaystyle\int_0^3 \sqrt{x+1}dx$
22. $\displaystyle\int_0^2 \frac{2x^3}{\sqrt{9+x^4}}\,dx$
23. $\displaystyle\int y^2 e^y dy$
24. $\displaystyle\int_0^{\pi/6} \cos^2 x\ dx$

25. Find the area of the region bounded by the graph of $y = |\cos x|$ and the x-axis from $x = -\pi$ to $x = \pi$.