Logic and Set Theory
PSU Math Relays 2018

- Answer each of the following on the answer sheet provided.
- Simplify each answer as far as possible.
- You may **not** use a calculator on this test.
- Notations: ϕ denotes the empty set. U denotes the universe of discourse. S^c denotes the complement of S in U. $|S|$ is the number of elements in a finite set S. $\vee$ means "or". $\wedge$ means "and". $\cap$ means "intersection". $\cup$ means "union". $\neg$ means "not". $\forall$ means "for all". $\exists$ means "there exists". $\in$ means "is an element of". $\subseteq$ means "subset of".

1. If $|A| = 23$, $|B| = 17$, and $|A \cup B| = 36$, then $|A \cap B| =$?

2. Let $S = \{x, \{x\}\}$. Determine which of the following assertions are correct.
   (a) $\phi \in S$ (b) $\phi \subseteq S$ (c) $\{x\} \in S$
   (d) $\{x\} \subseteq S$ (e) $\{x, \{x\}\} \in S$ (f) $\{x, \{x\}\} \subseteq S$

In problems 3–5 let the universe of discourse be $U = \{a, b, c, d, e, f, g, h\}$. Let $A = \{a, b, c, d, e\}$, $B = \{a, c, f, h\}$, and $C = \{d, e, g, h\}$.

3. Find $(A \cup C) \cap (B \cup C)$.
4. Find $A^c \cup B^c$.
5. Find $A - (B - C)$.

In problems 6–11, A, B, C are sets. Determine whether the statement is true (T) or false (F).

6. If $A \cup B = B$, then $A \subseteq B$.
7. If $A \in B$ and $B \in C$, then $A \in C$.
8. If $A \subseteq B$ and $B \in C$, then $A \in C$.
9. If $A \cap B \subseteq C$, then $A \subseteq C$ and $B \subseteq C$.
10. If $A \not\subseteq B$ and $C \subseteq B$, then $A \not\subseteq C$.
11. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

In problems 12–15, determine whether the statement is true or false.

12. If $x \in A$ and $A \not\subseteq B$, then $x \notin B$.
13. If $x \in A$ and $A \in B$, then $x \in B$.
14. If $x \notin B$ and $A \subseteq B$, then $x \notin A$.
15. If $x \notin A$ and $A \subseteq B$, then $x \notin B$.

In problems 16–19, compute the following truth table.

	p	q	$p \wedge (\neg q \vee \neg p)$
16.	T	T	
17.	T	F	
18.	F	T	
19.	F	F	

In problems 20–21, assume that p and r are true and q is false. Determine whether each proposition is true or false.

20. $\neg(q \to r)$

21. $(p \to (q \vee r)) \wedge ((\neg r \vee q) \to \neg p)$

In problems 22–24, determine whether each statement is true or false. The domain of discourse is $\boldsymbol{R}$, the set of real numbers.

22. $\forall x((x < 1) \to (x^2 < x))$

23. $\forall x \exists y((x < y) \to (x^2 < y^2)$

24. $\exists x \forall y((y > x) \to (y > x^2 - 1))$.

In problems 25–27 let p represent the statement, "If $x > 2$ then $x^2 > 4$." Consider the following statements:

(a) If $x > 2$ then $x^2 \leq 4$.

(b) If $x^2 > 4$ then $x > 2$.

(c) If $x^2 \leq 4$ then $x \leq 2$.

(d) If $x \leq 2$ then $x^2 \leq 4$.

(e) For some $x > 2$, $x^2 \leq 4$.

25. Which of these statements is equivalent to the negative of p?

26. Which of these statements is equivalent to the converse of p?

27. Which of these statements is equivalent to the contrapositive of p?

In problems 28–30 determine whether the statement is a tautology.

28. $(p \to q) \leftrightarrow (\neg p \vee q)$

29. $((p \to q) \wedge (\neg p \to q)) \leftrightarrow \neg q$

30. $((p \to r) \wedge (q \to r)) \leftrightarrow (p \vee q \to r)$