Notation and Definitions:

- $\gcd(a, b)$ denotes the *greatest common divisor* of the positive integers a and b.

- $\operatorname{lcm}(a, b)$ denotes the *least common multiple* of the positive integers a and b.

- If $\gcd(a, b) = 1$, we say a and b are *relatively prime*.

- $\varphi(n)$ is *Euler's phi function*, which is the number of positive integers less than or equal to n which are relatively prime to n.

- $a \equiv b(\bmod m)$ if $a - b$ is an integer multiple of m.

- $\lceil x \rceil$ denotes the "ceiling" of x, the smallest integer greater than or equal to x.

- $\lfloor x \rfloor$ denotes the "floor" of x, the greatest integer less than or equal to x.

- $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ denotes the binomial coefficient.

Instructions: Write your answer on the indicated line of the answer sheet **exactly as directed** in each problem. Only the answer sheet will be graded.

1. How many positive divisors does $2^3 \cdot 3^4 \cdot 5^3$ have?

2. List all positive divisors of 170 in **increasing order**.

3. Please find $\gcd(260, 126)$.

4. Please find $\operatorname{lcm}(154, 86)$.

5. What is the exponent on 3 in the prime factorization of 16200?

6. What is the exponent on 7 in the prime factorization of 135828?

7. Suppose that a, b are positive integers and we know that $\operatorname{lcm}(a, b) = 144$ and $a \cdot b = 3456$. Find $\gcd(a, b)$.

8. Please find the 21st prime.

Questions 9-11 refer to the following definition of a, b:
Suppose that a, b have prime factorizations
$$a = 2^4 \cdot 3^8 \cdot 5^2$$
$$b = 2^3 \cdot 3^7 \cdot 11^{10}$$

9. What is the exponent on 3 in the prime factorization of $\operatorname{lcm}(a, b)$?

10. What is the exponent on 2 in the prime factorization of $\gcd(a, b)$?

11. What is the exponent on 5 in the prime factorization of $\operatorname{lcm}(a, b)$?

12. What is the smallest possible positive value for c satisfying the equation below, assuming that x, y are arbitrary integers?
$$9x + 15y = c.$$

13. Please find $\gcd(840, 180)$.

14. Which of the numbers below is divisible by 3? **Write either "x" or "y" on the answer sheet.**

$$x = 452,313$$
$$y = 11,231,131$$

15. Which of the numbers below is divisible by 8? **Write either "x" or "y" on the answer sheet.**

$$x = 636,128$$
$$y = 3,423,426$$

16. Please calculate $\lfloor\sqrt{170}\rfloor$.

17. Please calculate $\lceil\sqrt{52}\rceil$.

18. Please calculate $\varphi(41)$.

19. Please calculate $\varphi(36)$.

20. Please calculate $\varphi(143)$.

21. What is the coefficient of a^3b^2 in the simplified expansion of $(a + 2b)^5$?

22. Please calculate

$$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}.$$

23. Please convert the base 2 (binary) number 110110 to base 10.

24. Please convert the base 10 number 587 to base 5.

25. What values of $x \in \{0,1,2,3,4,5,6\}$ satisfy $2x - 3 \equiv 1(\text{mod } 7)$?

26. What values of $x \in \{0,1,2,3,4,5\}$ satisfy $x^2 \equiv 4(\text{mod } 6)$?

27. Please calculate $57^{2017}(\text{mod } 8)$ (**answer as an integer x, $0 \le x \le 7$**)

28. Please calculate $89^{2017}(\text{mod } 10)$ (**answer as an integer x, $0 \le x \le 9$**)

29. There are integers a, b so that

$$23a + 16b = 1.$$

Please find the value of b.

30. Please find x so that $5x = 1(\text{mod } 18)$ (**answer as an integer x, $0 \le x \le 17$**)

Thank you for participating in the Pittsburg State Math Relays!