Marathon–2019

You may use a calculator. **Do not write on the test below but only on the plain paper provided.** Answers put on the form below will not be graded.

1. Find all of the roots of $x^3 - 10x^2 + 31x - 30 = 0$.

2. Suppose $L(0) = 1$ and that, for $n = 1, 2, 3, \ldots$, $L(n) = nL(n-1)$.

    (a) Calculate $L(1)$.
    (b) Calculate $L(2)$.
    (c) Calculate $L(7)$.

3. Suppose $\dfrac{1}{n(n+1)} = \dfrac{A}{n} + \dfrac{B}{n+1}$

    (a) Show that $\dfrac{1}{n+1} = A + \dfrac{A}{n+1} \cdot n$
    (b) Find the value of A.
    (c) Find the value of B.

4. $S_n = \dfrac{1}{1 \cdot (1+1)} + \dfrac{1}{2 \cdot (2+1)} + \cdots + \dfrac{1}{n \cdot (n+1)}$

    (a) Calculate S_1.
    (b) Calculate S_2.
    (c) Calculate S_{1000}.

5. (a) Expand and simplify $(x + x_1)(x + x_2)$.
    (b) Expand and simplify $(x + x_1)(x + x_2)(x + x_3)$.
    (c) What is the coefficient of x^{n-1} in $(x + x_1)(x + x_2)(x + x_3) \ldots (x + x_n)$? Don't be afraid to guess.
    (d) What is the constant term in $(x + x_1)(x + x_2)(x + x_3) \ldots (x + x_n)$? Don't be afraid to guess.

6. What is the constant term of $(x + 1)(x + 2)(x + 3) \ldots (x + n)$?

7. Prove that if x_1, x_2, and x_3 are the roots of $x^3 + 2x + 30 = 0$, then $x_1 + x_2 + x_3 = 0$.