

**Notation and Definitions:**

- $\gcd(a, b)$  denotes the *greatest common divisor* of the positive integers  $a$  and  $b$ .
- $\text{lcm}(a, b)$  denotes the *least common multiple* of the positive integers  $a$  and  $b$ .
- $a \equiv b \pmod{m}$  if  $a - b$  is an integer multiple of  $m$ .
- $\lceil x \rceil$  denotes the “ceiling” of  $x$ , the smallest integer greater than or equal to  $x$ .
- $\lfloor x \rfloor$  denotes the “floor” of  $x$ , the greatest integer less than or equal to  $x$ .

**Instructions:** Write your answer on the indicated line of the answer sheet **exactly as directed** in each problem. Only the answer sheet will be graded.

1. List all positive divisors of 78 in **decreasing order**.
2. How many positive divisors does 180 have?
3. Please find  $\gcd(18, 63)$ .
4. Please find  $\text{lcm}(12, 32)$ .
5. What is the exponent on 2 in the prime factorization of 1680?
6. What is the exponent on 5 in the prime factorization of 17325?
7. Suppose that  $a, b$  are positive integers and we know that  $\gcd(a, b) = 12$  and  $a \cdot b = 1488$ . Find  $\text{lcm}(a, b)$ .
8. Please find the 19th prime.

**Questions 9-11 refer to the following definition of  $a, b$ :**

Suppose that  $a, b$  have prime factorizations

$$a = 2^7 \cdot 3^9 \cdot 7^2$$

$$b = 2^5 \cdot 3^5 \cdot 11^3$$

9. What is the exponent on 2 in the prime factorization of  $\text{lcm}(a, b)$ ?
10. What is the exponent on 3 in the prime factorization of  $\gcd(a, b)$ ?
11. What is the exponent on 11 in the prime factorization of  $\text{lcm}(a, b)$ ?

**Questions 12-18 are true/false. You must write the complete words “True” or “False” for credit on the answer sheet. Writing just the letters “T” or “F” will result in no credit:**

12. **True or False:** 4,436,243 is divisible by 2.
13. **True or False:** 12,334,335 is divisible by 3.
14. **True or False:** 455,342,753,112 is divisible by 4.
15. **True or False:** 678,123,234 is divisible by 5.
16. **True or False:** 122,456,126 is divisible by 6.
17. **True or False:** 23,908,100 is divisible by 8.
18. **True or False:** 29,048,340 is divisible by 9.

19. Please find  $\gcd(12321, 4233)$ .
20. Please calculate  $\lfloor \sqrt{197} \rfloor$ .
21. Please calculate  $\lceil \sqrt{123} \rceil$ .
22. What values of  $x \in \{0, 1, 2, 3, 4, 5\}$  satisfy  $3x - 2 \equiv 1 \pmod{6}$ ?
23. What values of  $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$  satisfy  $x^2 \equiv 7 \pmod{9}$ ?
24. Please calculate  $13^{2019} \pmod{12}$  (**answer as an integer  $x$ ,  $0 \leq x \leq 11$** )
25. Please calculate  $11^{2019} \pmod{12}$  (**answer as an integer  $x$ ,  $0 \leq x \leq 11$** )
26. Please convert the base 10 number 264 to base 4.
27. Please convert the base 2 (binary) number 1011011 to base 10.
28. Please convert the base 10 number 157 to base 2 (binary).
29. What is the coefficient of  $a^2b^4$  in the simplified expansion of  $(2a + b)^6$ ?
30. Please calculate  $1 + 2 + 3 + 4 + 5 + \dots + 75$ . (the sum of the first 75 natural numbers)

Thank you for participating in the Pittsburgh State Math Relays!