MATH RELAYS 2019
PROBABILITY & STATISTICS

Place your answer on the appropriate blank of the answer sheet. Express each answer accurate to 3 decimal places.

For problems 1-7 consider the sample data set: 4, 1, 5, 8, 9, 10, 5

1.   Find the median.

2.   Find the range.

3.   Find the mode.

4.   Find the sample mean.

5.   Find the sample variance.

6.   Find the sample standard deviation.

7.   Find the standard error of the mean.

For problems 8-11, consider the following discrete distribution.

X	0	1	2	3	4
$P(X)$	0.1	0.1	0.1	0.1	0.6

8.   Find the mean of the distribution.

9.   Find the variance of the distribution.

10.   Find the mean of $Y = -2X + 3$.

11.   Find the variance of $Y = -2X + 3$

For problems 12 and 13, use the following information.

x	1	2	3	4	5
y	4.63	5.40	6.18	6.56	7.48

12.   Find the slope of the least square regression line.

13.   Find the y-intercept of the least square regression line.

14.   What is the predicted value of y when $x = 2.5$?

15.   Consider two events, A and B such that $P(A) = 0.39$ and $P(A \cap B') = 0.27$.
       Find $P(A' \cup B')$.

16.   Consider two events, A and B such that $P(A) = 0.35$, $P(B) = 0.50$, and
       $P(A' \cap B) = 0.30$. Find $P(A|B)$.

17.   Let A and B are two independent events such that $P(B) = 0.4$ and $P(A \cap B') = $
       0.2. Find $P[A \cap B]$.

18.   If somebody invests $10,000 at 2.0%, $8,000 at 1.5%, and $2,000 at 4%, then find
       the overall percentage yield. Give the answer as a percentage.

19.   Among the 10 candidates for four positions on a city council, 4 are Democrats, 4
       are Republicans, and 2 are Independents. In how many ways can the 5
       councilmen be chosen randomly so that 2 are Democrats, 2 are republicans, and 1
       is an Independent?

20.   In a lot of 10 light bulbs, there are 3 defective bulbs.   An inspector selects 2 bulbs
       one at a time without replacement and tests them. What is the probability that
       both bulbs are defective?

21.   Let a container has 4 black balls and 6 white balls. All of them are identical other
       than the color. Randomly select three balls **without** replacement. Find the
       probability of getting two black balls and one white ball.

22. Let $P(X=x) = c\left(\dfrac{1}{6}\right)^{x-1}$ for $x = 1, 2, ...$, and $P(X=x) = 0$ otherwise. Find the value of c.

23. Let $P(X=x) = \left(\dfrac{1}{2}\right)^{x}$ for $x = 1, 2, 3, ...$, and $P(X=x) = 0$ otherwise. Let $A = \{2, 5, 8, ...\}$. Find $P(A)$.

24. If the probability mass function of a random variable X is given by

    $f(x) = \left(\dfrac{1}{2}\right)^{x}$ for $x = 1, 2, 3, ...$, then find the $P(X \geq 5)$.

25. If the probability mass function of a random variable X is given by

    $f(x) = \left(\dfrac{1}{2}\right)^{x}$ for $x = 1, 2, 3, ...$, then find the $P(X > 4 \mid X > 1)$.

26. If the probability mass function of a random variable X is given by $f(x) = \dfrac{c}{x(x+1)}$ for $x = 1, 2, 3, ...$, then find the value of $P(X=1)$.