Set Theory and Logic

PSU Math Relays 2019

- There are 39 problems

- For each problem, place your answer in the appropriate blank of the answer sheet provided.

- Simplify each answer as much as possible.

- No calculators are allowed on the exam.

- Notations: $\emptyset$ denotes the empty set. U denotes the universe of discourse. S^c denotes the complement of S in U. $|S|$ is the number of elements in a finite set S. $\mathcal{P}(S)$ is the set of all subsets of a set S. $\vee$ means "or". $\wedge$ means "and". $\neg$ means "not". $\forall$ means "for all". $\exists$ means "there exists".

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1. If $|A| = 21$, $|B| = 17$, and $|A \cap B| = 3$, then $|A \cup B| = $?

2. What is the converse of $p \to q$?

3. What is the contrapositive of $q \to p$?

In problems 4-9 let $S = \{\emptyset, x\}$. State whether the following are true or false. Please write the word true or false, DO NOT use T or F.

4. $\emptyset \in S$

5. $\{x\} \in S$

6. $\emptyset \subseteq S$

7. $\{\emptyset\} \in S$

8. $\{\emptyset, x\} \subseteq S$

9. $x \subset S$

In problems 10-13 let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$.

10. How many elements does $\mathcal{P}(B)$ have?

11. How many functions are there from A to B?

12. How many one-to-one functions are there from A to B?

13. How many onto functions are there from B to A?

For problems 14-17 state whether the following are true or false. Please write the word true or false, DO NOT use T or F.

14. If $A \cap B \cap C = \emptyset$, then $B \cap C = \emptyset$.

15. If $A \subseteq B$, then $A^c \subseteq B^c$.

16. If $A \cup B = A$, then $A \subseteq B$.

17. If $A \cap B = \emptyset$, then $A \subseteq B^c$.

For problems 18-25 complete the following truth tables:

	p	q	$p \wedge \neg q$
18.	T	T	
19.	T	F	
20.	F	T	
21.	F	F	

	p	q	$(p \wedge q) \vee q$
22.	T	T	
23.	T	F	
24.	F	T	
25.	F	F	

For problems 26-28 let p represent the statement, "She is intelligent," and q the statement "She reads books." Write the following statements symbolically, using the symbols $p, q, \neg, \wedge, \vee$.

26. Either she is intelligent or she does not read books.

27. She is intelligent and she does not read books.

28. Neither is she intelligent nor does she read books.

For problems 29-31 let $\mathbb{R}$ be the universe of discourse. Are the following statements true or false? Please write the word true or false, DO NOT use T or F.

29. $\forall\, x \in \mathbb{R} \; \exists\, y \in \mathbb{R} : x < y$.

30. $\forall\, x \in \mathbb{R} \; \exists\, y \in \mathbb{R} : y^2 = x$.

31. $\forall\, x \in \mathbb{R} \; \exists\, y \in \mathbb{R} : y = x^2$.

For problems 31-36, let $U = \{-3, 0, \sqrt{2}, \sqrt{\pi}, 2^{\frac{1}{4}}\}$, $A = \{x \in U \mid x \text{ is an integer}\}$, and $B = \{x \in U \mid x^2 \text{ is an integer}\}$.

32. List the elements of A.

33. List the elements of B.

34. List the elements of $A \cap B$.

35. List the elements of $A \cup B$.

36. List the elements of $A \cap B^c$.

37. Lists the elements of $A - B$.

For problems 38-49 determine whether the statement is a tautology. Answer "yes" if the statement is a tautology and "no" if it is not a tautology.

38. $(q \wedge p) \vee (\neg p \wedge \neg q)$

39. $\neg(p \wedge \neg p)$