Trigonometry
PSU Math Relays 2019

- For each problem choose the correct answer and shade the corresponding letter completely on the answer sheet. Choose "(E) none" if no correct answer is given as a choice in (A), (B), (C), or (D).
- You may write on the test but only the answer sheet will be graded.
- **No calculator** is allowed on this test.
- All angles are given in radians unless they appear in degrees such as 30°.

1. Find the exact value of $\cos^2\left(\frac{\pi}{12}\right)$.

   (A) $\frac{2+\sqrt{3}}{4}$ (B) $\frac{1}{16}$ (C) $\frac{3}{16}$ (D) $\frac{\sqrt{3}}{4}$ (E) none

2. Convert $\frac{11\pi}{15}$ rad to degrees.

   (A) 264° (B) 132° (C) $264\pi^\circ$ (D) $132\pi^\circ$ (E) none

3. Find the period of the function $y = 5\sin(3x-2)$.

   (A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) none

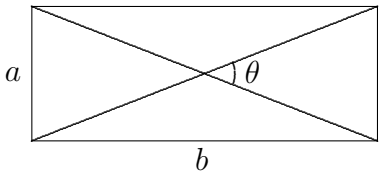
4. Let $\theta = \frac{\pi}{3}$. Evaluate $\sin\theta + \sin(2\theta) + \sin(3\theta) + \cdots + \sin(99\theta) + \sin(100\theta)$ exactly.

   (A) 0 (B) $-\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) none

5. In the rectangle shown in the figure below, two sides, a and b, and an angle between the two diagonals θ are given. Find $\sin\theta$.

   (A) $\frac{\sqrt{ab}}{a^2+b^2}$ (B) $\frac{\sqrt{ab}}{\sqrt{a^2+b^2}}$

   (C) $\frac{ab}{a^2+b^2}$ (D) $\frac{ab}{\sqrt{a^2+b^2}}$ (E) none



6. Find all the values of x between 0° and 360° such that $\sin x > \cos x$.

   (A) $45^\circ < x < 135^\circ$ (B) $45^\circ < x < 180^\circ$ (C) $45^\circ < x < 225^\circ$

   (D) $45^\circ < x < 270^\circ$ (E) none

7. An equilateral triangle is inscribed in an unit circle. What is the area of the triangle?

(A) $\frac{4\sqrt{3}}{5}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) none

8. If $(-2, 3)$ is a point on the terminal side of an angle θ in standard position, then $\csc\theta =$

(A) $-\frac{2}{3}$ (B) $-\frac{3}{2}$ (C) $\frac{\sqrt{13}}{3}$ (D) $\frac{3}{\sqrt{13}}$ (E) none

9. Solve the equation $\sin x = \cos(2x)$ for x in $\left(\frac{\pi}{2}, \pi\right)$.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$ (E) none

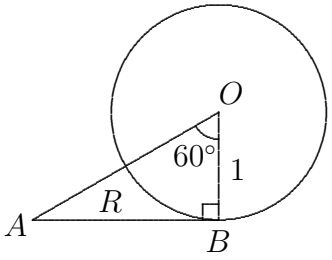
10. How many solutions does the equation $\cos^2 x = \frac{1}{5}$ have for $0 \le x \le 2\pi$?

(A) one (B) two (C) three (D) four (E) none

11. In the figure shown below, find the area of the region R which lies inside the right triangle OAB but outside the unit circle centered at O.

(A) $1 - \frac{\pi}{6}$ (B) $\sqrt{2} - \frac{\pi}{6}$ (C) $\sqrt{3} - \frac{\pi}{6}$

(D) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (E) none



12. $\sin^{-1}[\sin(210°)] =$

(A) 210° (B) 150° (C) 60° (D) 30° (E) none

13. Given $\cos x = \frac{1}{\sqrt{10}}$ and $\csc x = \frac{-\sqrt{10}}{3}$, find the exact value of $\tan x$.

(A) -3 (B) 3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$ (E) none

14. Find the exact value of $\cot\left(\frac{5\pi}{6}\right)$.

(A) 2 (B) -2 (C) $\frac{\sqrt{3}}{3}$ (D) $-\frac{\sqrt{3}}{3}$ (E) none

15. Convert $-225°$ to the radian measure.

(A) $-\frac{5\pi}{3}$ (B) $-\frac{4\pi}{3}$ (C) $-\frac{5\pi}{4}$ (D) $-\frac{6\pi}{5}$ (E) none

16. Simplify $\frac{1-\cos^2 t}{\sin^3 t}$.

(A) $\sec t$ (B) $\csc t$ (C) $\tan t$ (D) $\cot t$ (E) none

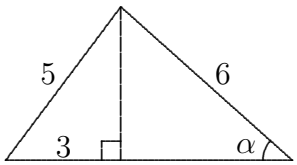
17. Find the exact value of $\sin\left(\frac{-4\pi}{3}\right)$.

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$ (E) none

18. In the figure shown below, find $\cot\alpha$.

(A) 1 (B) $\frac{\sqrt{5}}{2}$ (C) $\frac{\sqrt{5}}{3}$

(D) $\frac{2\sqrt{5}}{5}$ (E) none



19. Given $\sin x = \frac{2}{\sqrt{5}}$, find the exact value of $\cos 2x$.

(A) $\frac{2}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $-\frac{3}{5}$ (E) none

20. Simplify $\frac{\sin(2x)}{\cos^2 x}$.

(A) $\tan(2x)$ (B) $2\tan x$ (C) $\sec(2x)$ (D) $\sec x \tan x$ (E) none

21. Solve the equation $\sin x = \sin(2x)$ for x in $\left(0, \frac{\pi}{2}\right)$.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$ (E) none

End of exam